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Static pair correlation function of electrons around an infinite mass positively charged impurity in one-and two-component classical and quantum rare hot plasmas

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Abstract . The static pair correlation function $g^{\pm}(r)$, of electrons around an infinite mass positively charged impurity in one and two component classical and quantum rare hot plasmas has been computed by making use of the recently suggested frequency and wave vector dependent complete dielectric function $g^{\pm}(r)$ for quantum plasma are quite different from the classical plasma particularly at low temperatures and near the impurity

Keywords Classical and quantum plasma, static pair correlation function, dielectric function

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1. Introduction

Two component rare plasma is encountered in the ionosphere, intergalactic space, laboratory and doped semiconductors at appropriate density and temperature. When only one of the components is mobile and the other is assumed to be immobile, the plasma is called one component plasma. When thermal de-Broglie wavelength $\left(\lambda_{th} = \frac{h}{\sqrt{2\pi m k_B T}}\right.$, where m is the mass of the mobile component, h is the Planck's constant and k_B is the Boltzmann's constant) of the mobile component cannot be ignored in comparison with the mean interparticle distance, $2r_s$ (r_s being the radius of a sphere assigned to each particle, the plasma is referred to as quantum plasma. Frequency and wavevector dependent dielectric function yields complete dynamics of the plasma. The problem of determining static pair correlation function $g^{\pm}(r)$ of the plasma [1,2] around a charged impurity introduced in the system is directly related to its dielectric function and is therefore important as it determines the extent of screening of the charged particle. If the mass of the impurity corresponds to a positron, the value of static pair correlation function at the site of the positron, determines essentially the positron annihilation rate [3-6].

The static pair correlation function $g^{\pm}(r)$, of electrons around an infinite mass positively charged impurity in one component quantum rate hot plasma had been calculated using the expression for quantum dielectric function at zero frequency [7,8], taking into account the contributions up to $\hbar^2$ terms [9]. The expression thus, obtained, was therefore approximate and so were the computed values of $g^{\pm}(r)$. In the present work, $g^{\pm}(r)$ of electrons around an infinite mass positively charged impurity have been computed using the complete dielectric function *i.e.*, to all orders in $\hbar^2$ and exact expression for $g^{\pm}(r)$, both for one component and two component plasmas.

2. Mathematical formalism

The static pair correlation function $g^{\pm}(r)$ of electrons around an impurity of finite mass M, is given as [1, 2]

$$g^{\pm}(\boldsymbol{r}) = 1 + \frac{1}{(2\pi)^3} \int_{-\infty}^{\infty} f(\boldsymbol{q}) \exp(-i\boldsymbol{q}.\boldsymbol{r})\, d\boldsymbol{q}\,. \tag{1}$$

When the density of positive charge introduced is small, $f(\boldsymbol{q})$ is given as :

$$f(\boldsymbol{q}) = -\frac{z}{n}\,\mathrm{Re}\left(\frac{1}{\varepsilon^{--}(\boldsymbol{q}, \hbar q^2/2M)} - 1\right) + \frac{z}{n}\frac{2}{\pi}\int_0^{\infty} d\omega\, \mathrm{Im}\left(\frac{1}{\varepsilon^{--}(\boldsymbol{q},\omega)}\right)$$

$$\times P\left(\frac{\hbar q^2/2M}{\omega^2 - (\hbar q^2/2M)^2}\right), \tag{2}$$

where $\varepsilon^{--}(\boldsymbol{q},\omega) = \varepsilon_1(\boldsymbol{q},\omega) + i\varepsilon_2(\boldsymbol{q},\omega)$ is the complex dielectric function of the plasma, n is the number density of the electrons in the system and z is a positive integer. $\omega\ (= \hbar q^2/2M)$ and $\boldsymbol{q}$ are the angular frequency and wavevector respectively.

For a weakly degenerate two component plasma, the dielectric function is given as [10, 11].

$$\varepsilon_1^0(q,\omega) = 1 + \frac{\sqrt{2}\,\omega_{p_\pm}^2}{q^2 v_\pm^2}\left(\frac{m_\pm v_\pm}{hq}\right)\Bigg[\exp\left(-\left(\frac{\omega}{\sqrt{2}qv_\pm} + \frac{hq}{2\sqrt{2}\,m_\pm v_\pm}\right)^2\right)$$

$$\times\left(\frac{\omega}{\sqrt{2}qv_\pm} + \frac{hq}{2\sqrt{2}\,m_\pm v_\pm}\right)\times\left\{1 + \frac{1}{3}\left(\frac{\omega}{\sqrt{2}qv_\pm} + \frac{hq}{2\sqrt{2}\,m_\pm v_\pm}\right)^2 + \dots\right\}$$

$$-\exp\left(-\left(\frac{\omega}{\sqrt{2}qv_\pm} - \frac{hq}{2\sqrt{2}\,m_\pm v_\pm}\right)^2\right)\times\left(\frac{\omega}{\sqrt{2}qv_\pm} - \frac{hq}{2\sqrt{2}\,m_\pm v_\pm}\right)$$

$$\times\left\{1 + \frac{1}{3}\left(\frac{\omega}{\sqrt{2}qv_\pm} - \frac{hq}{2\sqrt{2}\,m_\pm v_\pm}\right)^2 + \dots\right\}\Bigg], \tag{3}$$

$$\varepsilon_2^0(q,\omega)=\sqrt{\frac{\pi}{2}}\,\frac{\omega_{p_\pm}^2}{q^2v_\pm^2}\left(\frac{m_\pm v_\pm}{hq}\right)\left[\exp\left(-\left(\frac{\omega}{\sqrt{2}qv_\pm}-\frac{hq}{2\sqrt{2}\,m_\pm v_\pm}\right)^2\right)\right.$$

$$\left.-\exp\left(-\left(\frac{\omega}{\sqrt{2}qv_\pm}+\frac{hq}{2\sqrt{2}\,m_\pm v_\pm}\right)^2\right)\right], \tag{4}$$

where $\omega_{p_\pm}\left(=\sqrt{4\pi n_\pm e^2/m_\pm}\right)$, $v_\pm\left(=\sqrt{k_BT_\pm/m_\pm}\right)$, $n_\pm$ and $T_\pm$ respectively are angular frequency, thermal velocity, density and temperature of the positive and negative components of the plasma. m_+ is the mass of the positive component of the plasma and m_-, the mass of negative component of the plasma is equal to the mass of an electron.

These expressions reduce to the corresponding classical expressions when h is put equal to zero [10] *i.e.*,

$$\varepsilon_1^{cl}(q,\omega)=1+\frac{\omega_{p_\pm}^2}{q^2v_\pm^2}-\frac{\omega_{p_\pm}^2}{q^2v_\pm^2}\,\frac{\omega^2}{q^2v_\pm^2}\exp\left(-\left(\omega^2/2q^2v_\pm^2\right)\right)$$

$$\times\left\{1+\frac{1}{3}\left(\frac{\omega^2}{2q^2v_\pm^2}\right)+\frac{1}{10}\left(\frac{\omega^2}{2q^2v_\pm^2}\right)^2+\ldots\ldots\right\}, \tag{5}$$

$$\varepsilon_2^{cl}(q,\omega)=\sqrt{\frac{\pi}{2}}\,\frac{\omega_{p_\pm}^2}{q^2v_\pm^2}\,\frac{\omega}{qv_\pm}\exp\left(-\left(\omega^2/2q^2v_\pm^2\right)\right). \tag{6}$$

When only one of the components is considered to be mobile, the expressions given by eqs. (3) and (4) for quantum plasma and (5) and (6) for classical plasma reduce to the corresponding expressions for one component quantum and classical plasmas [10].

For infinite mass positively charged impurity introduced in the two component plasma, *i.e.* for $M=\infty$, expression (2) for $f(\boldsymbol{q})$ reduces to.

$$f(\boldsymbol{q})=f_\pm(\boldsymbol{q})=-\frac{z}{n_\pm}\,\mathrm{Re}\left(\frac{1}{\varepsilon(\boldsymbol{q},0)}-1\right). \tag{7}$$

The imaginary part of the dielectric function vanishes when $\omega\to 0$. Since we are interested in infinite mass impurity, we need zero frequency dielectric function $\varepsilon(\boldsymbol{q},0)$ for two component quantum and classical plasmas, which are as follows :

$$\varepsilon^0(q,0)=1+\frac{\sqrt{2}\,\omega_{p_\pm}^2}{q^2v_\pm^2}\left(\frac{m_\pm v_\pm}{hq}\right)\left[2\,\exp\left(-\left(hq/2\sqrt{2}\,m_\pm v_\pm\right)^2\right)\right.$$

$$\left.\times\left(\frac{hq}{2\sqrt{2}\,m_\pm v_\pm}\right)\times\left\{1+\frac{1}{3}\left(\frac{hq}{2\sqrt{2}\,m_\pm v_\pm}\right)^2+\ldots\ldots\right\}\right]. \tag{8}$$

$$\varepsilon^{(1)}(q,0) = 1 + \frac{\omega^2_{p_\pm}}{q^2 v_\pm^2}. \tag{9}$$

If only one component is mobile, say negative, the expressions (8) and (9) would contain only the negative sign terms.

3. Results and discussion

Substituting expressions for dielectric function for one component plasma and two component plasma in expression (1), the static pair correlation function $g^\pm(r)$ have been accurately computed by properly summing up the series occurring in the expressions for different values of r expressed in units of r_s.

In Figure 1, classical and quantum values of $g^\pm(r)$ at a realistic density 1.0×10^{15} cm^{-3} when $M = \infty$, for one component plasma at 30K and 77K and for two component plasma with $m_+ = 5m_-$ at 30K and 77K (where the difference between the quantum and classical plasma is significant), are illustrated for r expressed in terms of r_s. As is evident, values of $g^\pm(r)$ for two component plasma for both the temperatures is always greater than the corresponding values for one component plasma upto $r \sim 0.7\, r_s$, beyond which they start oscillating about $g^\pm(r) = 1$ and the oscillations persist till $r \sim 3r_s$. At any given temperature and for both the plasmas, classical value is always higher than the quantum value. This result turns out to be same at other higher temperatures. For a given plasma : two component or one component, $g^\pm(r)$ decreases with the increase in temperature. The difference between classical and quantum

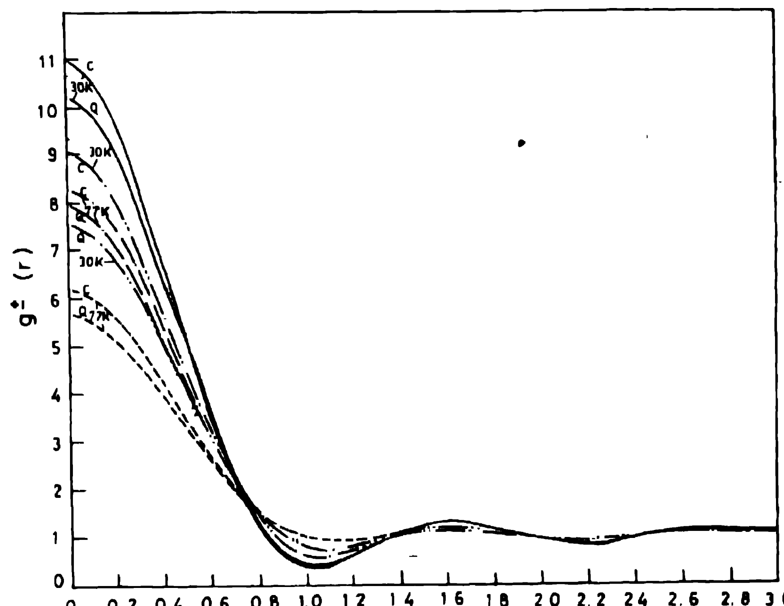

Figure 1. Calculated values of static pair correlation function, $g^\pm(r)$, of electrons around on infinite mass impurity with distance r expressed in terms of r_s for one and two component quantum and classical rare hot plasmas having number density equal to 1.0×10^{15} cm^{-3} at temperatures 30K and 77K. Two component classical and quantum plasma for $m_+ = 5m$ at 30 K are shown by (———) and at 77K by (— — — ——). One component classical and quantum plasma at 30K are shown by (— —) and at 77K by (— — —).

values of $g^{\pm}$ (r) decreases as temperature increases though the decrease is more for one component plasma *i.e.* this difference at 30K is larger than that at 77K. This is because of the fact that dual nature of electrons becomes increasingly important as one goes to lower temperatures.

The computed variations of static pair correlation function $g^{\pm}$ (r), of electrons around an infinite mass impurity in two component plasma at realistic density $n_{\pm} = 1.0 \times 10^{15}$ cm^{-3}, temperature $T_{\pm} = 30$ K (where the quantum mechanical effect will be reflected) for different masses of the positive component of the quantum plasma and classical plasma with r are shown in Figure 2. Since the expression for the two component classical dielectric function for an infinite mass impurity is independent of the mass of the positive component of the plasma m_+, $g^{\pm}(r)$ is same for all m_+. For the quantum two component plasma, with the increase in m_+, value of $g^{\pm}(r)$ increases for small values of r. Difference between the value of $g^{\pm}(r)$ at $m_+ = 1m_-$ and $m_+ = 5m_-$ is much greater than at $m_+ = 5m_-$ and $m_+ = 10m_-$, though the classical $g^{\pm}(r)$ is greater than all the three quantum values for small values of r. For both quantum and classical plasma, $g^{\pm}(r)$ decreases till $r \sim 0.7\, r_s$ and then starts oscillating about $g^{\pm}(r) = 1$. The oscillatory behaviour dampens with the increase in r. In the inset of the figure are shown the oscillations for the quantum plasma ($m_+ = 1m_-$ and $m_+ = 5m_-$) and classical plasma till $r \sim 3r_s$.

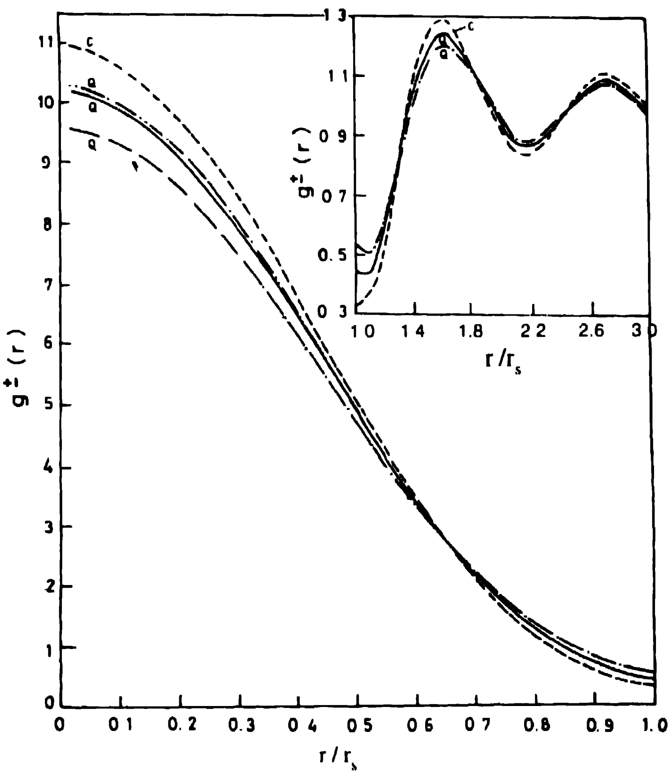

Figure 2. Variation of static pair correlation function, $g^{\pm}$ (r), of electrons around an infinite mass impurity with distance r expressed in terms of r_s for two component plasma having number density $n_{\pm} = 1.0 \times 10^{15}$ cm^{-3} at $T_{\pm} = 30$K for different values of m_+ shown by : $m_+ = 1m_-$ (—.—.—), $m_+ = 5m_-$ (————) and $m_+ = 10m_-$ (—..—..—). $g^{\pm}$ (r) for classical plasma is independent of m_+ and is shown by (— — — —). In the inset are shown the details of $g^{\pm}$ (r) for larger values of r.

Lastly, in Figure 3, we have plotted the ratio R of the quantum mechanical plasma $g^{\pm}(r)_Q$ to the classical plasma $g^{\pm}(r)_{cl}$ with different values of r, expressed in terms of r_s at density 1.0×10^{15} cm^{-3} and T = 30K for one component plasma (which are quite different from the one calculated using approximations in the expressions of $\varepsilon(q, \omega)$, $g^{\pm}(r)$ and computations) and two component plasma with $m_+ = 1m_-$, $m_+ = 5m_-$ and $m_+ = 10m_-$. For any given form of plasma, there is a peak at $r = 1r_s$ which is sharpest for $m_+ = 1m_-$ and the peak starts broadening with the increase in m_+ in two component plasma. After $r \geq 1.3r_s$, R shows oscillatory behaviour and approaches a constant value $R = 1$.

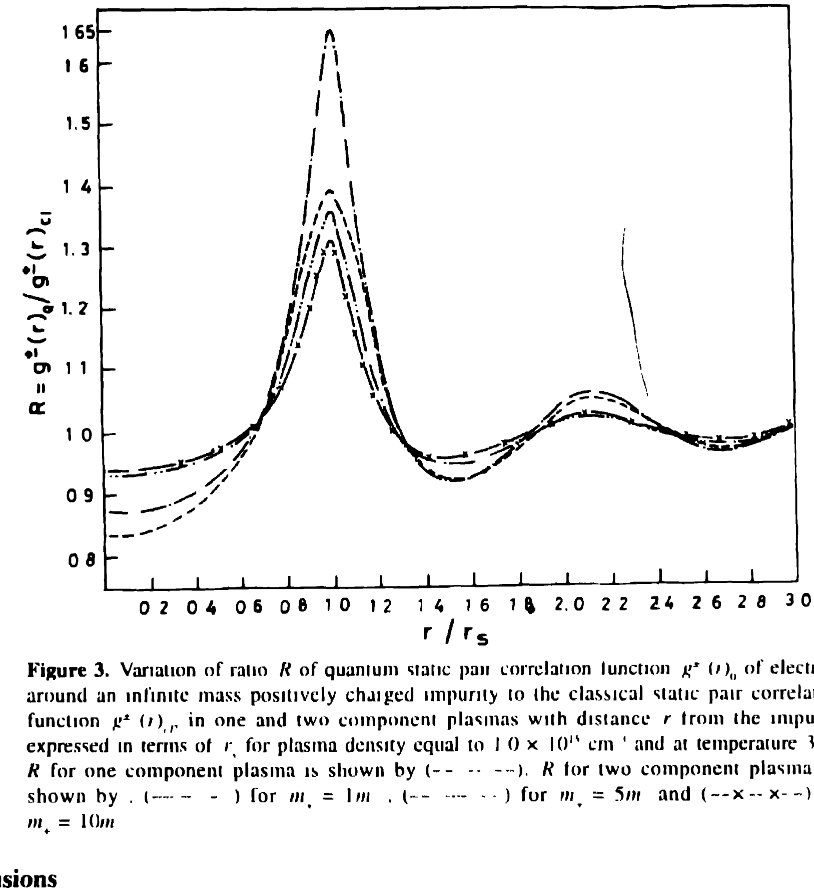

Figure 3. Variation of ratio R of quantum static pair correlation function $g^{\pm}(r)_Q$ of electrons around an infinite mass positively charged impurity to the classical static pair correlation function $g^{\pm}(r)_{cl}$ in one and two component plasmas with distance r from the impurity expressed in terms of r_s for plasma density equal to 1.0×10^{15} cm^{-3} and at temperature 30K. R for one component plasma is shown by (-- -- --). R for two component plasma are shown by . (---- -- -) for $m_+ = 1m_-$, (-- ---- - -) for $m_+ = 5m_-$ and (--x-- x- -) for $m_+ = 10m_-$

4. Conclusions

From our study, we conclude that static pair correlation functions of electrons around an infinite mass positively charged impurity in both one component and two component quantum plasmas are quite different from the classical cases. Suitable experiment in a laboratory plasma or a doped semiconductor at appropriate density and temperature may be performed to observe the onset of quantum effect in one and two component plasmas around a heavy positively charged impurity.

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